

# Derivative Table

## Basic Properties of Differentiation

$$\frac{d}{dx}[af(x) \pm bg(x)] = a \frac{d}{dx}f(x) + b \frac{d}{dx}g(x)$$

$$\frac{d}{dx}[f(x) \cdot g(x)] = \left[ \frac{d}{dx}f(x) \right] \cdot g(x) + f(x) \cdot \left[ \frac{d}{dx}g(x) \right]$$

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \cdot \left[ \frac{d}{dx}f(x) \right] - f(x) \cdot \left[ \frac{d}{dx}g(x) \right]}{g^2(x)}$$

$$\frac{d}{dx} \left[ \frac{1}{f(x)} \right] = - \frac{\frac{d}{dx}[f(x)]}{f^2(x)}$$

$$\frac{d}{dx}\{f[g(x)]\} = \frac{d}{dg}[f(g)] \cdot \frac{d}{dx}[g(x)]$$

## Common Derivatives

### Algebraic Functions

$$\frac{d}{dx}a = 0$$

$$\frac{d}{dx}[x] = 1$$

$$\frac{d}{dx}[ax] = a$$

$$\frac{d}{dx}(x^a) = ax^{a-1}$$

$$\frac{d}{dx}(a^x) = a^x \ln(a), \quad a > 0$$

$$\frac{d}{dx}(x^x) = x^x(1 + \ln x)$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}[y^n] = ny^{n-1} \left( \frac{dy}{dx} \right)$$

$$\frac{d}{dx}[\log_a y] = \frac{1}{\ln a} \cdot \frac{1}{y} \cdot \frac{dy}{dx}$$

$$\frac{d}{dx}(y^z) = zy^{z-1} \left( \frac{dy}{dx} \right) + (\ln y) \cdot y^z \left( \frac{dz}{dx} \right)$$

$$\frac{d}{dx} \left( \frac{1}{x} \right) = -\frac{1}{x^2}$$

$$\frac{d}{dx} \left( \frac{1}{x^2} \right) = -\frac{2}{x^3}$$

$$\frac{d}{dx} \left( \frac{1}{x^n} \right) = -\frac{n}{x^{n+1}}$$

$$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2 \cdot \sqrt{x}}, \quad x > 0$$

$$\frac{d}{dx}(\sqrt[3]{x}) = \frac{1}{3 \cdot \sqrt[3]{x^2}}$$

$$\frac{d}{dx}(\sqrt[n]{x}) = \frac{1}{n \cdot \sqrt[n]{x^{n-1}}}$$

$$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = -\frac{1}{2 \cdot \sqrt{x^3}}$$

$$\frac{d}{dx}\left(\frac{1}{\sqrt[3]{x}}\right) = -\frac{1}{3 \cdot \sqrt[3]{x^4}}$$

$$\frac{d}{dx}\left(\frac{1}{\sqrt[n]{x}}\right) = -\frac{1}{n \cdot \sqrt[n]{x^{n+1}}}$$

<b>Logarithms</b>
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$$\frac{d}{dx}(\ln x) = \frac{1}{x}, \quad x > 0$$

$$\frac{d}{dx}(x \cdot \ln x) = \ln x + 1$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}, \quad a > 0 \text{ and } a \neq 1$$

$$\frac{d}{dx}\left(\frac{1}{\ln x}\right) = -\frac{1}{x(\ln x)^2}$$

$$\frac{d}{dx}\left(\frac{1}{x \ln x}\right) = -\left[\frac{\ln x + 1}{(x \ln x)^2}\right]$$

$$\frac{d}{dx}\left(\frac{1}{\log_a x}\right) = -\frac{1}{x \cdot \ln a \cdot (\log_a x)^2}$$

## Trigonometric Functions

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \sec^2 x = \frac{1}{\cos^2 x}$$

$$\frac{d}{dx} \cot x = -\operatorname{csc}^2 x = -\frac{1}{\sin^2 x}$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \operatorname{csc} x = -\operatorname{csc} x \cot x$$

## Inverse Trigonometric Functions

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}, \quad |x| < 1$$

$$\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}, \quad |x| < 1$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \cot^{-1} x = -\frac{1}{1+x^2}$$

$$\frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}, \quad |x| > 1$$

$$\frac{d}{dx} \operatorname{csc}^{-1} x = -\frac{1}{x\sqrt{x^2-1}}, \quad |x| > 1$$

## Hyperbolic Functions

$$\frac{d}{dx} \sinh x = \cosh x$$

$$\frac{d}{dx} \cosh x = \sinh x$$

$$\frac{d}{dx} \tanh x = 1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\frac{d}{dx} \coth x = 1 - \coth^2 x = -\operatorname{csch}^2 x$$

$$\frac{d}{dx} \operatorname{sech} x = -\tanh x \operatorname{sech} x$$

$$\frac{d}{dx} \operatorname{csch} x = -\coth x \operatorname{csch} x$$

## Inverse Hyperbolic Functions

$$\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{x^2 + 1}}$$

$$\frac{d}{dx} \cosh^{-1} x = \frac{1}{\sqrt{x^2 - 1}}$$

$$\frac{d}{dx} \tanh^{-1} x = \frac{1}{1 - x^2}, \quad |x| < 1$$

$$\frac{d}{dx} \coth^{-1} x = \frac{1}{1 - x^2}, \quad |x| < 1$$

$$\frac{d}{dx} \operatorname{sech}^{-1} x = -\frac{1}{x\sqrt{1 - x^2}}, \quad x > 0$$

$$\frac{d}{dx} \operatorname{csch}^{-1} x = -\frac{1}{x\sqrt{x^2 + 1}}, \quad x > 0$$

## Inverses of Polynomials

$$\frac{d}{dx} \left( \frac{1}{x+1} \right) = -\frac{1}{(x+1)^2}$$

$$\frac{d}{dx} \left[ \frac{1}{(x+1)^2} \right] = -\frac{2}{(x+1)^3}$$

$$\frac{d}{dx} \left[ \frac{1}{(x+1)^n} \right] = -\frac{n}{(x+1)^{n+1}}$$

## Inverses of Square Roots of Polynomials

$$\frac{d}{dx} \left( \frac{1}{\sqrt{x+1}} \right) = -\frac{1}{2 \cdot \sqrt{(x+1)^3}}$$

$$\frac{d}{dx} \left[ \frac{1}{\sqrt[3]{x+1}} \right] = -\frac{1}{3 \cdot \sqrt[3]{(x+1)^4}}$$

$$\frac{d}{dx} \left[ \frac{1}{\sqrt[n]{x+1}} \right] = \frac{-1}{n \cdot \sqrt[n]{(x+1)^{n+1}}}$$